

# CALCULATION OF THE FIELD OF STATIC CHARGES IN FLUIDIZED-BED DEVICES

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UDC 621.317.1:537.22

A procedure is proposed for calculating electrostatic fields of charged particles of the working material in fluidized-bed devices by using experimental data on the volumetric distribution of charged particles in the apparatus.

An estimate of the danger of static electrification and breakdown in the treatment of dielectric materials in a fluidized bed in the final analysis involves the calculation of the electrostatic field in the apparatus. Commonly used devices have grounded metal frames. Therefore, the electrostatic field is calculated in the volume within the inner surface of the device. The problem consists in calculating the electrostatic field from a given volumetric distribution of the charge of the working material of the bed. It reduces to the integration of Poisson's equation and is solved for specified boundary conditions which are determined from the conditions of the technological process and the structural features of the given device.

At the present time is impossible to calculate the density distribution of the working material over the volume of the apparatus and its time fluctuations [1, 2]. This makes it impossible to calculate the volumetric charge density and its distribution from a known value of the average charge of the particles of the working material. It thus becomes necessary to determine the volumetric charge distribution and its fluctuations experimentally in each particular case.

We have measured the indicated parameters in a fluidized-bed device. A schematic diagram of the arrangement for measuring the charge distribution in a fluidized bed of dry granular polymeric materials is shown in Fig. 1. The device is conical in shape, is made of plastic, and has the following dimensions:  $d_1 = 0.19$  m,  $d_2 = 0.4$  m,  $H = 1.0$  m. The apex angle of the cone  $\alpha \approx 7^\circ$ . The fraction of the cross-sectional area of the metal distribution grid which is clear is  $\delta = 0.203$ . Atmospheric air with controlled humidity was chosen as a fluidizing agent. The metal grounded cap was lowered in order to approach production conditions as closely as possible inside the shell of the apparatus. The working material was mark PSB polystyrene beads 1.05–2.5 mm in diameter.

Polystyrene was chosen as the working material because it is only slightly hygroscopic and is not readily contaminated. These factors are particularly important from the point of view of the reproducibility of the results of measurements of charges of static electricity in the fluidized bed.

An induction method was used to measure the volumetric distribution of the charges of the particles of the working material in the fluidized bed. Induction transducers recorded the number of passing particles, their charges, and local velocities [3, 4]. This method eliminates the appreciable distortion of the hydrodynamics of the process caused by the insertion of the transducer into the bed, gives results which are convenient to process, and makes relatively modest demands on the quality of the electrical insulation.

The transducer 2 (Fig. 1) consists of a square metal frame filled with foil 0.1 mm thick. The foil is covered with a layer of insulating lacquer over which is applied an Aquadag screen which is grounded to eliminate the effect of the external field. The effective height of the transducer for which a passing particle induces a charge on the frame turned out to be 0.15 mm, permitting its use for small porosities of the bed. The dimensions of the frame were determined by starting from the requirement that the device not measure charges of several particles simultaneously, counting them erroneously as the charge of one particle [3].

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Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 29, No. 3, pp. 419–425, September, 1975.  
Original article submitted August 20, 1973.

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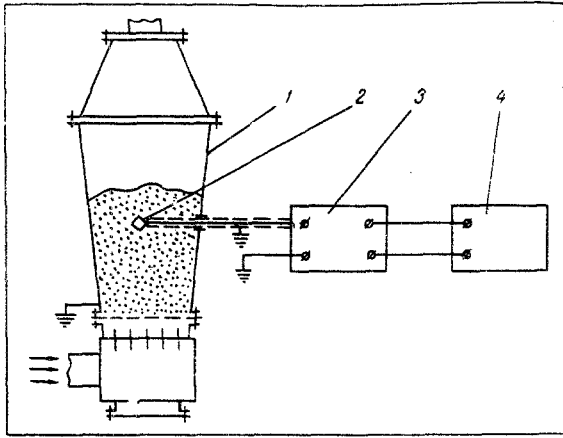


Fig. 1

Fig. 1. Schematic diagram of fluidized-bed device with measuring instruments. 1) Device; 2) transducer; 3) U4-1 constant and alternating voltage amplifier; 4) EO-7 oscillograph.

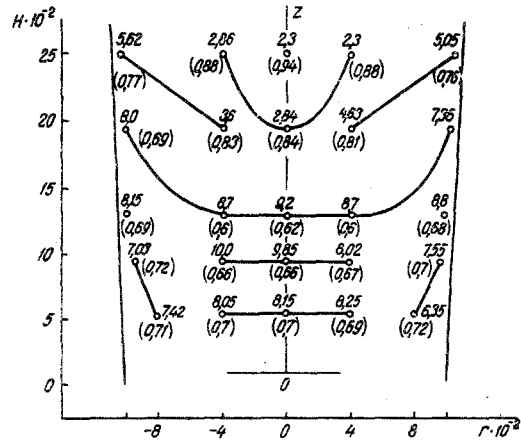


Fig. 2

Fig. 2. Volumetric charge-density distribution in a fluidized-bed device. Data of bed:  $w_{air} = 2.9$  m/sec,  $d_p = 2.2$  mm,  $\varphi = 70\%$ . The numbers in parentheses are the porosity of the bed; the numbers without parentheses are the charge densities  $\rho \cdot 10^{-4}$  C/m<sup>3</sup>; H and r are in meters.

In order that the probability of simultaneously finding two or more particles in the volume in which the incident charged particles will be recorded by the transducer be no more than 5% of the probability of finding one particle, it is necessary that

$$Qk \leq 0.1. \quad (1)$$

For particles of the working material having a diameter  $d_p = 2.2$  mm the length of an inside edge of the frame of the transducer was 2.6 mm. By placing the transducer frame in the x, y, and z directions all the particles at the transducer location moving in any direction can be taken into account. For example, the concentration of particles along the x axis is given by

$$k_x = \frac{N_x}{\omega_p S_f}. \quad (2)$$

The true concentration of particles is

$$k = k_x + k_y + k_z. \quad (3)$$

Using the calibration curve of the transducer the average charge of particles along the x, y, z axes and the volumetric charge density in the bed are determined by the expression

$$\rho = kq_{av}. \quad (4)$$

The method of measurement consists in the following: The transducer is placed at the point of measurement (25 such points were chosen in the device) (Fig. 2) and the signal from the transducer is amplified by amplifier 3 and recorded by the oscillograph 4. The oscillograph screen is photographed with a motion-picture camera and the film is used to determine the number of pulses per unit time and the pulse width and height. These are input data for compiling a program to calculate the concentration, velocities, and average charges of the particles. The results of the measurements were processed on an ODRA-1204 computer. On the basis of the results obtained a picture of the volumetric charge distribution in the device under study was constructed (Fig. 2). It is the input for calculating the electrostatic field in the apparatus. Since this particular conical device has an apex angle  $\alpha \approx 7^\circ$ , it can be treated as a cylinder, simplifying the calculation. In this case it is convenient to write Poisson's equation in cylindrical coordinates.

Because of the symmetry of the problem we assume that the potential does not depend on  $\theta$  and that  $\partial^2 U / \partial \theta^2 = 0$ . Then we have

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} = -\frac{\rho}{\epsilon_0}. \quad (5)$$

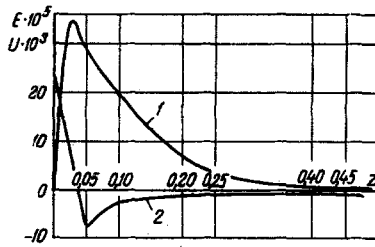


Fig. 3

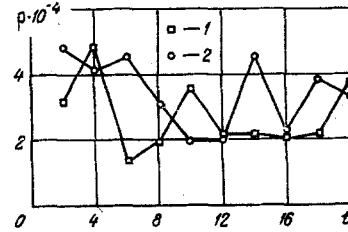


Fig. 4

Fig. 3. Field intensity and potential as functions of height of bed. 1)  $U = f(z)$  at  $r = 0$ ; 2)  $E = f(z)$  at  $r = 0.1$  m.  $E$  is in V/m and  $U$  is in V.

Fig. 4. Time dependence of charge density; 1)  $\rho_1(t)$  and 2)  $\rho_2(t)$  at the center and at the wall of the apparatus at a height  $H = 0.195$  m.  $\rho \cdot 10^{-4}$  is in  $C/m^3$  and  $t$  is in sec.

In solving Eq. (5) it is assumed that the radius of the apparatus  $R = 1$  and the height  $H$  is arbitrary except that  $R \ll H$ . Equation (5) is solved for specified boundary conditions: 1)  $z = 0$ ,  $U = 0$ ; 2)  $r = R = 1$ ,  $U = 0$  on the surface of the cylinder; 3)  $z = \infty$ ;  $U = 0$ ; 4) the apparatus is considered as an infinite cylinder. To solve the problem we expand the function  $\rho(r, z)$  in a series of Bessel functions,

$$\rho(r, z) = \sum_{n=1}^{\infty} a_n(z) J_0(x_n r), \quad (6)$$

where the expansion coefficients are

$$a_n(z) = \frac{2}{[J_1(x_n)]^2} \int_0^1 r \rho(r, z) J_0(x_n r) dr, \quad (7)$$

and the  $x_n$  are the roots of the Bessel function. Then the expansion of the potential will be

$$v(r, z) = \sum_{n=1}^{\infty} b_n(z) J_0(x_n r). \quad (8)$$

It satisfies Eq. (5) and therefore the coefficients are found in terms of the known  $a_n(z)$  from the equation

$$b_n''(z) - x_n^2 b_n(z) = -\frac{a_n(z)}{\epsilon_0}. \quad (9)$$

We consider the special case when  $\rho(z)$  varies exponentially. This case corresponds to the distribution  $\rho(z) = \rho_0 e^{-\beta z}$  in the main area of the fluidized bed. Then

$$a_n(z) = \frac{2\rho_0 e^{-\beta z}}{x_n J_1(x_n)},$$

and Eq. (9) takes the form

$$b_n''(z) - x_n^2 b_n(z) = -\frac{2\rho_0 e^{-\beta z}}{\epsilon_0 x_n J_1(x_n)}. \quad (10)$$

The solution of the equation has the following form:

$$b_n(z) = C_2 e^{-x_n z} + \frac{2\rho_0 e^{-\beta z}}{\epsilon_0 (x_n^2 - \beta^2) x_n J_1(x_n)}.$$

Using the initial condition  $b_n(0) = 0$  we obtain, finally,

$$b_n(z) = \frac{2\rho_0}{\epsilon_0 (x_n^2 - \beta^2) x_n J_1(x_n)} [e^{-\beta z} - e^{-x_n z}] \quad (11)$$

or

$$V(r, z) = \sum_{n=1}^{\infty} \frac{2\rho_0 J_0(x_n r)}{\epsilon_0 (x_n^2 - \beta^2) x_n J_1(x_n)} [e^{-\beta z} - e^{-x_n z}]. \quad (12)$$

The calculation can be simplified by determining the average value of the potential over a horizontal cross section of the bed by integrating Eq. (8). Then we obtain

$$\tilde{U}(z) = \sum_{n=1}^{\infty} b_n(z) \frac{2\pi \int_0^1 r J_0(x_n r) dr}{\pi}.$$

The integral is expressed in terms of the first-order Bessel function and we obtain, finally,

$$\tilde{U}(z) = 2 \sum_{n=1}^{\infty} \frac{J_1(x_n)}{x_n} b_n(z).$$

Substituting  $b_n(z)$  into the solution of Eq. (10), we obtain

$$\tilde{U}(z) = \frac{4\rho_0}{\epsilon_0} \sum_{n=1}^{\infty} \frac{e^{-\beta z} - e^{-x_n^2 z}}{x_n^2 (x_n^2 - \beta^2)}$$

or

$$\tilde{U}(z) = \frac{4\rho_0}{\epsilon_0} \left\{ e^{-\beta z} \sum_{n=1}^{\infty} \frac{1}{x_n^2 (x_n^2 - \beta^2)} - \sum_{n=1}^{\infty} \frac{e^{-x_n^2 z}}{x_n^2 (x_n^2 - \beta^2)} \right\}. \quad (13)$$

If  $\rho$  is constant over the whole volume of the apparatus  $\beta = 0$  and

$$\tilde{U}(z) = \frac{4\rho_0}{\epsilon_0} \left\{ \sum_{n=1}^{\infty} \frac{1}{x_n^4} - \sum_{n=1}^{\infty} \frac{e^{-x_n^2 z}}{x_n^4} \right\}. \quad (14)$$

By using Eqs. (12), (13), and (14) the potential at any point inside the apparatus can be calculated from the charge-density distribution. By processing the experimental data on the charge distribution in the main area of the fluidized bed (Fig. 2)  $\rho(z)$  can be written in the form

$$\rho(z) = 14 \cdot 10^{-4} e^{-5.45z}.$$

Figure 3 shows the calculated potential and field-intensity distributions over the height of the bed at the wall of the apparatus. They enable us to judge the maximum value of the field intensity which determines the condition for the breakdown of the medium and the production of an electrical discharge. Figure 4 shows the time dependence of the charge density at the point  $r = 0$ ,  $H = 0.195$  m and at  $r = 0.1$  m,  $H = 0.195$  m.

The graphs show that in practice the field intensity in the apparatus does not reach the breakdown value. However, this does not prove that there are no electrical discharges in the apparatus. An oscillographic investigation shows that the particles of the working material discharge to the frame of the apparatus by gas discharge. Since the charges of the particles of the bed are measured in the presence of electrical discharges in the apparatus, they are limited by the field intensity which maintains a gas discharge in the bed. It always turns out to be smaller in magnitude than the initial field intensity at which breakdown begins [6]. Therefore, the field intensity calculated from the experimental values of the particle charges does not reach its breakdown value. The nature of the time pulsations of charge density about its average value shown in Fig. 4 is of great practical importance. This is explained by the fact that the field intensity which leads to the production of an electrical discharge is characterized by the instantaneous rather than by the average value. The charge-density pulsations characterize the error in estimates of the danger of static electrification in apparatus based on the average value of the field intensity in the bed.

#### NOTATION

$\epsilon_0$	is the dielectric constant of air;
$\rho$	is the volumetric charge density;
$R$	is the radius of apparatus;
$H$	is the height of apparatus;
$Q$	is the volume in which charged particles will be recorded by transducer;
$k$	is the concentration of particles at point of measurement;
$N$	is the number of pulses per unit time on oscillogram;
$w_p$	is the velocity of particles at point of measurement;
$S_p$	is the area of active cross section of frame;
$q_{av}$	is the average charge of particles;
$V$	is the potential in a uniform medium;

$J_0(x_n r)$	is the zero-order Bessel function;
$J_1(x_n r)$	is the first-order Bessel function;
$x_n$	are the roots of Bessel function;
$r, z$	are the arbitrary quantities;
$a_n(z), b_n(z)$	are the coefficients in differential equation;
$E$	is the field intensity;
$t$	is the time;
$d_p$	is the particle diameter;
$w$	is the air velocity;
$\varphi$	is the humidity of air, %.

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